Lecture 22 Pressure in a liquid

Atmospheric pressure is $1.01 * 10^5 Pa = 14.7 lb/in^2 = 29.92 in\ Mercury = 1013.25 mbar$. On the weather maps you see the pressure in millibars (mbar). The average pressure at the earth surface is about $1013.25mbar$, while a "high" on the weather map indicates a region of high pressure, while a "low" on the weather map indicates a region of low pressure. In high pressure regions there is a slow flow of air from high altitudes to earth and in the low pressure regions, there is a flow or air from earth upwards. The latter tends to cause rain as the air at earth is more humid and it cools as it rises, leading to condensation of water and hence rain. This evening (Thursday 27th) there is a high over the midwest which has barometric pressure 1026mbar, and there is a low at the edge of this high that has barometric pressure 1010mbar. The lowest recorded pressure in the eye of hurricane was Hurricane Wilma at $882mbar$.

Atmospheric pressure is due to the weight of the air in the atmosphere. We need to understand how to calculate the pressure in fluids (gases and liquids) and how the pressure changes when a liquid flows and in gases when we change the temperature. It is interesting to note that the force on the human body due to air pressure is of order 1 ton. However there is a balancing pressure inside the body, so we don't get squashed. The human body is designed to handle this sort of pressure.

Pressure in liquids

Liquids only resist hydrostatic pressure, which means that they have to be confined in all directions. Perhaps the most useful and maybe puzzling fact is that when a liquid is stationary, pressure is transmitted to all surfaces of the container of the liquid. The simplest case is where the fluid has no externally applied pressures, except the weight of gravity. In that case, we can find the weight of fluid(e.g. water) which acts on any surface which is at a depth of h below the surface of the fluid. The force due to the weight of water acting on an area A at depth h below the surface of the water is, $F = \rho A h g$ the total pressure at this depth is the pressure due to this force, plus the pressure due to the weight of air above it, we thus have,

$$P = P_0 + \rho g h \tag{1}$$

Since for water $\rho = 10^3 kg/m^3$, the pressure at depth h is given by,

$$P = 1.01 * 10^5 Pa + 9.81 * 10^3 h Pa \tag{2}$$

The human body can adapt to the higher pressures which occur at greater depths, however problems occur due to the fact that nitrogen and oxygen uptake in the body are strongly altered by pressure so that special air mixes are required. The record for a scuba dive is 313m, where the water pressure is of order 30 times atmospheric pressure.

Hydraulic systems use the fact that an enclosed fluid transmits an externally applied pressure to all parts of the container, this is Pascal's Principle. A really useful aspect of Pascal's principle is that we can design hydraulic systems which can transmit pressure and hence force around corners, all we need is a sealed pipe. We can also amplify force using hydraulics, so that heavy objects can be lifted using a relatively small force. The pressure change must be the same, so we have, $\Delta P_1 = \Delta P_2$, so that $F_1/A_1 = F_2/A_2$, which implies that

$$F_2 = \frac{A_2}{A_1} F_1 \tag{3}$$

This can be used as a force amplifier! Of course nothing is for free so that if we want to raise a heavy object a distance Δh_2, we have to do the same amount of work, so that,

$$F_1 \Delta h_1 = F_2 \Delta h_2 \tag{4}$$

Measuring pressure

Manometers and barometers are devices for measuring pressure, P of a gas usually. In a barometer, a tube is evacuated and placed upside down in a fluid. The pressure in the gas remaining in the tube is $P_g << P$. Due to the ambient pressure, P, the fluid rises in the tube until the column of fluid (e.g. mercury) in the tube is high enough to balance the pressures, ie,

$$P = P_g + \rho g h \tag{5}$$

This has to be corrected for capillary forces which are important in thin tubes and which we will discuss later. That only offsets the zero of the column and does not affect the calculation. Consider the case of a mercury column in a perfectly evacuated tube so that $P_g = 0$. The density of mercury is, $\rho = 13.595 * 10^3 kg/m^3$, so at atmospheric pressure we have,

$$1.01 * 10^5 Pa = 0 + 13.595 * 10^3 kg/m^3 * 9.81 m/s^2 h \tag{6}$$

Solving for the height of the column we find that $h = 0.76m = 29.9in$. A manometer is even simpler, a column of fluid is open to the atmosphere and a gas pressure is maintained in a bulb at the other end of the tube. The location of the interface between the gas and the liquid at the closed end is monitored. A blood pressure monitor does not actually measure blood pressure directly. Instead it measures the pressure required to cut off blood circulation to your arm. The blood circulation is cut off using air pressure and the air pressure is measured using a barometer.

The buoyancy force

One of the earliest recorded hallelujah moments in science was the discovery of buoyancy by Archimedes who lived 287-212BC and probably studied with the followers of Euclid (325 265BC).

To understand the buoyancy force, imagine a beaker of water which is not moving. Now consider a small sphere of water in the beaker. This little sphere of water has weight $Mg = \rho_w V g$. However since the water is not moving, there must be a force which opposes the weight of the water and stops it from falling under gravity. This opposing force is the buoyancy force B. It is clear that we must have $B = Mg$ for our little sphere of water. However Archimedes generalized this observation to a general principle: *The buoyancy force of a body in a fluid is equal to the weight of the fluid which the body displaces.* This is a very useful and general principle which has a variety of surprising uses.